The theory of electromagnetic field motion.
10. Energy of electromagnetic field motion

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Paper considers dependence of the electromagnetic field energy motion on its velocity using plate electric capacitor as an example. On the basis of requiring the conformity of electromagnetic field properties and some other field equation properties, associated with its motion, to the laws of special relativity theory, the general equations for the field energy and the energy flux were obtained for moving electric and magnetic components. The results obtained allow make a conclusion on equality of electron mass, derived from the momentum equation and that derived from the relativistic energy equation, eliminating thereby the known contradiction. In addition to that the results allow make a conclusion that all relativistic mechanics basic laws which are valid for a substance, are also valid for electromagnetic field.

10.1. Introduction

In previous papers of the present cycle, in particular [1, 2], we have described some of electron properties. However there is one more paradox not considered earlier, the contradiction of the classical electromagnetic field theory, connected with electron motion, therefore it is necessary to study again the electromagnetic field properties, more particularly, to the energy of its motion.

Apparently, in the majority of the monographies devoted to the classical electromagnetic field theory, the problem of electromagnetic electron mass is mentioned to some extent. We select two of such monographies. This problem and, in particular, Kaufman hypothesis that all electronic mass is electromagnetic, and is considered by Lorentz in detail [3]. The same problem is mentioned by Feynman in [4] based on more modern positions. The problem as Feynman noted, consists in that the electromagnetic mass of electron, calculated on the basis of electromagnetic electron momentum, is not equal to the mass obtained from electron energy equation. This is internal contradiction of the classical electromagnetic field theory. The problem cannot be solved by the quantum theory, as Feynman underlines. In [4] the description of attempts is presented to eliminate the contradiction by existence of the internal forces which nature is distinct from electromagnetic, and unreliability of these attempts is underlined.

Actually the contradiction is deeper than Feynman tells about it: the standard method of calculation of electromagnetic mass of electron and, hence, energy and mass of electromagnetic field leads to the violation of the energy and momentum conservation laws. Let`s show this by example of calculation of energy in charged plate capacitor. Since our ultimate goal is not only criticism of modern views, but also obtaining new results which are free from existing contradictions, we make some preliminary general observations.

10.2. The electric capacitor

In the present paper we consider electric capacitor placed in inertial frame of reference as the example of a unique source of electromagnetic field. If there are more than one field source each of sources must be considered separately and irrespective of others according to the superposition principle, stated in [5].

We use concepts of intrinsic field and intrinsic velocity.

Generally the electromagnetic field has two components, electric and magnetic. In case of a single field source, namely it is this case is under consideration, there is an inertial frame of reference where there is only one of the components, electric or magnetic. Let`s name this component as intrinsic field of the source, or simply intrinsic field. This is the field in intrinsic frame of reference in which the source is motionless. Velocity of the intrinsic frame of reference relative to laboratory frame of reference we name as intrinsic velocity of the field source or, abstracting from the field source, but always meaning its existence, as intrinsic velocity of electromagnetic field. In the case of noninertial intrinsic frame of reference it is necessary, as it is generally accepted, to consider electromagnetic field at each point in instantly accompanying inertial frame of reference. Then the conclusions obtained below can be extended also to cases of noninertial frame of reference.

Let's remember some known properties of electromagnetic field invariants I_1 and I_2:

$$I_1 = c^2B^2 - E^2, \tag{10.1}$$

$$I_2 = \mathbf{BE} = 0, \tag{10.2}$$

where B - is magnetic field induction, E - electric field strength, and c - electromagnetic constant (velocity of light in vacuum). It should be noted that invariant I_2 in equation (10.2) in case of a single field source is always equal to zero, because this case vectors **B** and **E** are orthogonal. If invariant I_1 in equation (10.1) is positive, the intrinsic field is the magnetic field, if I_1 is negative the intrinsic field is the electric one. We designate intrinsic values of velocity, values of magnetic and electric field V_0, B_0 and E_0 respectively.

Let's proceed to calculation of electromagnetic mass of the charged capacitor. As rest mass m_0 and energy W are related to each other by simple known relation

$$W = \frac{m_0 c^2}{\sqrt{1 - V^2/c^2}}, \qquad (10.3)$$

where V is the velocity, calculation of electromagnetic mass is reduced to calculation of electromagnetic energy of the moving capacitor.

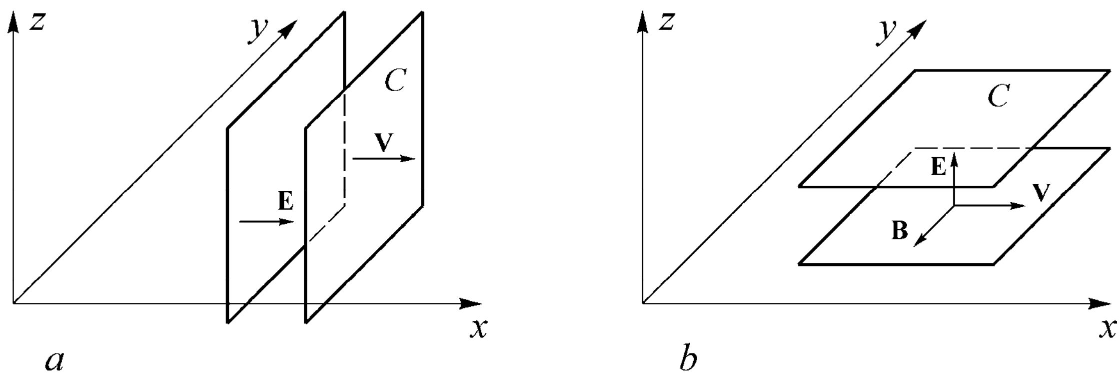

Fig. 10.1. Plate capacitor *C* moves along axis *x* at velocity **V**.
a is the case when direction of velocity and **E** vector are parallel to each other;
b is the case when direction of velocity and a electric field strength vector are perpendicular.

Plate capacitor C (fig. 10.1) with uniform electric field **E** moves in laboratory frame of reference xyz at a velocity **V**. In drawing 10.1*a the* direction of velocity and electric field strength vector **E** are parallel, and in drawing 10.1*b* they are perpendicular. Energy of capacitor plate material the obviously submits to the equation (10.3), therefore let exclude from consideration this energy and check up whether the electromagnetic field energy of the capacitor submits to the equation (10.3).

Equation in numerator of (10.3) is the rest energy. Rest energy W_0 of electric field of the capacitor, as known, is defined by equation

$$W_0 = \frac{\varepsilon_0 E_0^2 L_0}{2}, \tag{10.4}$$

where E_0 is the intrinsic electric field of the capacitor, ε_0 is electric constant and L_0 - the volume occupied with electric field of the capacitor in intrinsic frame of reference.

Rest energy in (10.3) is equal to numerator of this equation m_0c^2. Substituting in (10.3) instead of m_0c^2 equation for rest energy W_0 (10.4), we obtain:

$$W = \frac{\varepsilon_0 E_0^2 L_0}{2\sqrt{1 - V^2/c^2}}. \tag{10.5}$$

Such a replacement is valid. Let`s place our charged capacitor in a black box. Equation (10.3) is valid from the point of view of the special relativity theory (SRT) for a black box irrespective of the nature of the energy inside. Hence, equation (10.5) is also valid for the charged capacitor, and it is not depends on orientation of the black box and, hence, on orientation of electric field inside the box.

We have obtained equation (10.5) for total energy of the charged capacitor having in laboratory frame of reference velocity V. We have obtained this equation, proceeding from the general SRT laws. Now we obtain equation for the field energy of the capacitor, proceeding from known laws of the electromagnetic field theory.

When vectors $\mathbf{V}$ and $\mathbf{E}_0$ are parallel (fig. 10.1*a*), electric field $\mathbf{E}$, as follows from Lorentz's transformations for electromagnetic field, in laboratory frame of reference is equal to:

$$\mathbf{E} = \mathbf{E}_0, \tag{10.6}$$

and the magnetic field is absent.

In the case when vectors **V** and $\mathbf{E}_0$ are perpendicular (fig. 10.1*b*), the electric field **E** is equal to:

$$\mathbf{E} = \frac{\mathbf{E}_0}{\sqrt{1 - V^2/c^2}}, \tag{10.7}$$

and magnetic field **B** is equal:

$$\mathbf{B} = \frac{\frac{1}{c^2}[\mathbf{V}\mathbf{E}_0]}{\sqrt{1 - V^2/c^2}}. \tag{10.8}$$

Volume L which is occupied by fields E and B, is always defined by the formula:

$$L = L_0\sqrt{1 - V^2/c^2}, \tag{10.9}$$

since the transverse sizes at its motion are unchanged, and the longitudinal size decreases, as presented in (10.9).

Then energy of the capacitor electric field $W_{\parallel}'$ for its longitudinal motion is equal to the product of energy density by volume L:

$$W_{\parallel}' = \frac{\varepsilon_0 E^2 L}{2} = \frac{\varepsilon_0 E_0^2 L_0 \sqrt{1 - V^2/c^2}}{2}. \tag{10.10}$$

Here equations (10.6) and (10.9) are taken into account.

In the case of transverse motion the energy of capacitor electromagnetic field $W_{\perp}'$ is equal to the sum of electric field and magnetic field energy:

$$W_{\perp}' = \frac{\varepsilon_0 E^2 L}{2} + \frac{B^2 L}{2\mu_0} = \frac{\varepsilon_0 L}{2}\left(E^2 + c^2 B^2\right), \tag{10.11}$$

where μ_0 is magnetic constant, and the equality $1/\mu_0 = \varepsilon_0 c^2$ has also been taken into consideration.

Substituting in (10.11) values E, B and L from (10.7), (10.8) and (10.9) accordingly, we obtain after simple transformations:

$$W'_{\perp} = \frac{\varepsilon_0 E_0^2 L_0 \left(1 + V^2/c^2\right)}{2\sqrt{1 - V^2/c^2}}. \qquad (10.12)$$

Comparison (10.10) and (10.12) to (10.5) gives that $W'_{\parallel} < W < W'_{\perp}$. It means that if our black box with charged capacitor inside is differently orientated, we obtain different black box energy values and, hence, different values of momentum that means violation of fundamental physical laws i.e. energy conservation and momentum laws.

Now it becomes clear the reason of discrepancy between electromagnetic mass of classical electron and that derived from relativity theory.

Let's consider the possible reasons of contradiction equations (10.10) and (10.12) to the energy conservation law. These equations were derived on the basis of faultless laws of the relativity theory and electromagnetism, and their rejection means contradiction to the modern electromagnetic field theory.

There is just one shag to it: equations for energy electric and a magnetic field are obtained in the classical theory *provided that the fields are immovable*. This condition is never expressed explicitly, but always it is meant implicitly. Except for enough numerous and unsuccessful attempts of definition of electromagnetic mass of electron, anybody, as far as we know, did not try to calculate electric and magnetic field energy depending on their velocity. Paradox related to electromagnetic mass of electron is considered in [4] in most details.

10.3. Energy of moving electromagnetic field

Let's make one preliminary remark before proceed to calculation the energy of moving electromagnetic field.

Electromagnetic laws for vacuum take symmetric form relative to electric and magnetic fields, if electric field value is characterized by its strength E, and magnetic field value by the product cB. Thus from the equations magnetic constant μ_0 disappears which is free of physical meaning, whereas in equation for magnetic field energy density value ε_0 is appeared. In particular, so has occurred in equation (10.11). At such an approach ε_0 value is necessary to consider as electromagnetic constant not as electric one, the second electromagnetic constant along with electromagnetic constant c.

As we have shown above, equations (10.10) and (10.12) do not coincide with (10.5) what contradicts to the energy conservation law. To match these equations with equation (10.5) which is obviously valid, we enter unknown matching functions $f_1\ (V)$ and $f_2\ (V)$ such that double equality must be held

$$W'_{\parallel} f_1\left(V\right) = W = W'_{\perp} f_2\left(V\right) \qquad (10.13).$$

Let's find functions $f_1(V)$ and $f_2(V)$.

For the case on fig. 10.1*a* when **E** and **V** are parallel, we multiply the equation (10.10) by $f_1\ (V)$ and equate the obtained equation to the equation (10.5):

$$\frac{\varepsilon_0 E_0^2 L_0 \sqrt{1-V^2/c^2}}{2} f_1(V) = \frac{\varepsilon_0 E_0^2 L_0}{2\sqrt{1-V^2/c^2}}, \qquad (10.14)$$

whence

$$f_1\left(V\right) = \frac{1}{1-V^2/c^2}. \qquad (10.15)$$

Similarly for the case of perpendicularity of **E** and **V** (fig. 10.1*b*), we multiply the equation (10.12) by $f_2\ (V)$ and equate the obtained equation to the equation (10.5). As a result we obtain

$$\frac{\varepsilon_0 E_0^2 L_0 \left(1+V^2/c^2\right)}{2\sqrt{1-V^2/c^2}} f_2(V) = \frac{\varepsilon_0 E_0^2 L_0}{2\sqrt{1-V^2/c^2}}, \qquad (10.16)$$

whence

$$f_2(V)=\frac{1}{1+V^2/c^2}. \tag{10.17}$$

Thus, substituting in (10.10) and (10.12) functions $f_1(V)$ and $f_2(V)$ from (10.15) and (10.17), we come in all cases to equation for electromagnetic field energy (10.5). This equation is valid also for any direction of electric field since it can be decomposed in two components that are parallel and perpendicular to the direction of motion, and for every separate component equation (10.5) is valid.

All calculations presented above used electric capacitor as an example. It has been made only for clearness. Capacitor plates submit to all laws of relativistic mechanics and consequently they can be excluded from consideration. Certainly, instead the capacitor the infinitesimal element of the field, for which all conclusions are valid, can be chosen, and then summation on all volume of a field of any configuration is made, in particular, for a dot charge (electron), all results will also be valid. At last, if replacement E_0 by cB_0 is made as it was mentioned above, the equation (10.5) will be valid also for magnetic field:

$$W=\frac{\varepsilon_0 c^2 B_0^2 L_0}{2\sqrt{1-V^2/c^2}}. \tag{10.18}$$

From (10.5) and (10.18) taking into account (10.9) we obtain more usual form of the equation for electric field energy density w_e

$$w_e=\frac{\varepsilon_0 E_0^2}{2\left(1-V^2/c^2\right)} \tag{10.19}$$

and for magnetic field w_m

$$w_m=\frac{\varepsilon_0 c^2 B_0^2}{2\left(1-V^2/c^2\right)}. \tag{10.20}$$

If intrinsic field is electric, it is necessary choose the formula (10.19) (since in this case $cB_0 = 0$), if the field is magnetic the formula (10.20) must be chosen (in this case $E_0 = 0$). It should be noted that the equation (10.19) expresses *total field energy* of the capacitor independently on that in the laboratory frame of reference generally exists not only electric, but also magnetic field. Similar remark can be made and in relation to the equation (10.20).

Let's express energy density through the components of electromagnetic field E and B in laboratory frame of reference. With that end in view, considering as an example electromagnetic field with intrinsic electric field E_0, we represent energy density w as consisting of two components:

$$w_e = w_{e\|} + w_{e\perp}, \tag{10.21}$$

where $w_{e\|}$ is the energy density of electric field parallel component $E_{\|}$, and $w_{e\|}$ is the energy density of perpendicular component of electric field $E_{\perp}$. Using equation (10.19) for (10.21) we can write:

$$w_e = \frac{\varepsilon_0 E_{0\|}^2}{2\left(1 - V^2/c^2\right)} + \frac{\varepsilon_0 E_{0\perp}^2}{2\left(1 - V^2/c^2\right)}, \tag{10.22}$$

where $E_{0\|}$ and $E_{0\perp}$ are accordingly parallel and perpendicular components of intrinsic field E_0.

From the equations (10.6) both (10.7) we obtain for $E_{0\|}$ and $E_{0\perp}$ following relations:

$$E_{0\|} = E_{\|}, \quad E_{0\perp} = E_{\perp}\sqrt{1 - V^2/c^2}, \tag{10.23}$$

where $E_{\|}$ and $E_{\perp}$ are accordingly parallel and perpendicular components of electric field in laboratory frame of reference.

Substituting (10.23) in (10.22), after simplification we obtain for a case when intrinsic field is electric field:

$$w_e = \frac{\varepsilon_0 E_{\|}^2}{2\left(1 - V^2/c^2\right)} + \frac{\varepsilon_0 E_{\perp}^2}{2}. \tag{10.24}$$

Similarly it is possible to obtain equation for intrinsic magnetic field, but we use replacement E by cB what we have already used earlier

$$w_m = \frac{\varepsilon_0 c^2 B_{\parallel}^2}{2\left(1 - V^2/c^2\right)} + \frac{\varepsilon_0 c^2 B_{\perp}^2}{2}, \tag{10.25}$$

where $B_{\parallel}$ and $B_{\perp}$ are accordingly parallel and perpendicular components of magnetic field in laboratory frame of reference.

Attracting is that the second term $w_{e\perp}$ in (10.24) does not contain B although we have entered it into the equation (10.11) in an explicit form, but equation (10.24) is valid for total energy density of the moving capacitor since it is obtained from equation (10.19) for total energy density. It does not tell that all energy is contained only in electric component of electromagnetic field, it only points that it is impossible to divide electromagnetic field energy as single whole of the same source in the same volume into electric and magnetic component, such a division can only be conditional.

Let's find equation for $w_{e\perp}$ into which B enters in explicit form. With that end in view we return to the second of equalities (10.13). Let substitute in it $W'_{\perp}$ from the equation (10.11) and $f_2(V)$ from (10.17), and then divide the obtained equation by volume L and as a result we obtain equation for energy density of electric field perpendicular component:

$$w_{e\perp} = \frac{\varepsilon_0 \left(E_{\perp}^2 + c^2 B^2\right)}{2\left(1 + V^2/c^2\right)}. \tag{10.26}$$

This formula contains both electric field E and magnetic field B in explicit form. The bottom index in field B was omitted since the magnetic field in this case is secondary and cannot have longitudinal component. Analyzing (10.26), it is necessary be careful, because variable B in this equation is not independent value but is a function of electric field component $E_{\perp}$ and velocity of this field V_e (see (2.3) [6], and also equation (10.32) hereinafter).

We remind that all equation presented above were obtained for the case when there is only one field source, electric or magnetic. If velocity of the source is equal to zero all equations for energy density become simpler and are reduced to usual classical form.

10.4. Energy flux

Let's consider energy flux of moving field.

Let's find energy flux value $w_e V_e$ from (10.24) for an important case when the electric field longitudinal component of $E_{\|}$ is equal to zero. Substituting velocity $V_e = c^2 B/E$ in (10.24) and omitting the bottom index at $E_{\perp}$, we obtain:

$$w_e V_e = \frac{\varepsilon_0 E^2}{2} V_e = \frac{\varepsilon_0 c^2 BE}{2}. \qquad (10.27)$$

It should be noted that the right part (10.27) is equal to the Poynting vector multiplied by factor k=1/2 (see equation 8.1 [2]). Factor k=1/2 in all cases when there is obviously expressed source of field, electric or magnetic, and inertial frame of reference in which this source is motionless. The factor k=1 when such a source does not exist or the field loses connection to it. As an example may be electromagnetic wave, energy flow from direct current source into load by two-wire line or coaxial cable, electromagnetic field in electron core wall [2], etc.

The same result can be obtained if instead electric field motion we investigate magnetic field motion which is perpendicular to the electric field, the value of Poynting vector will be defined by the same equation (10.27).

It is also noted that in longitudinal motion of the field (field source) it is absolutely impossible to use Poynting vector as energy flux characteristics.

10.5. Properties of electric and magnetic field motion equations

Let's consider some additional curious properties of the obtained equations concerning electric and magnetic field motion together with their sources.

Let's find equation for intrinsic velocity of electric field source (capacitor).

From (10.26) follows:

$$w_e V = \frac{\varepsilon_0 \left(c^2 B^2 + E^2 \right)}{2\left(1 + V^2/c^2\right)} V . \qquad (10.28)$$

In the numerator (10.28) components are rearranged. Equating (10.27) and (10.28) we obtain:

$$\left(c^2 B^2 + E^2 \right) V = c^2 BE \left(1 + V^2/c^2 \right). \qquad (10.29)$$

Solving equation (10.29) relative to intrinsic velocity V as unknown variable, we find that:

$$V = c \frac{c^2 B^2 + E^2 \pm \left(c^2 B^2 - E^2 \right)}{2cBE}. \qquad (10.30)$$

It should be noted that in numerator (10.30) in parentheses the equation for I_1 invariant (10.1) is written down. As it is known, this equation is positive if intrinsic field is magnetic, and is less than zero, if intrinsic field is electric one.

From two velocity values V in (10.30) in the case of field source containing material elements, for example, capacitor plate material, the physical meaning has only value $V \le c$. This condition is valid if sign before brackets in (10.30) chosen by a following rule. If invariant of electromagnetic field in parentheses of equation (10.30) more than zero (intrinsic field is magnetic), we choose «minus» sign, if it is less than zero (intrinsic field is electric) sign «plus» is used.

The equation (10.30) generalizes equations (2.8) and (2.9) we have obtained earlier in [6] for velocity of electric and a magnetic field respectively.

Equation (10.29) is remarkable by its symmetry relative to E and cB. It is obtained for the case of intrinsic electric field, but it is equally applicable also to the case of intrinsic magnetic field. Let`s solve it at first to find electric field, and then to find magnetic field. After not too bulky transformations we obtain:

$$E = \frac{c^2 B}{2V}\left[1 + \frac{V^2}{c^2} \pm \left(1 - \frac{V^2}{c^2}\right)\right]. \qquad (10.31)$$

Equation (10.31) contains two roots of the equation (10.29). If sign «+» chosen before parentheses we obtain solution for case of intrinsic electric field and its velocity V_e:

$$E = \frac{c^2 B}{V_e}. \qquad (10.32)$$

Equation (10.32) is in accordance with the equation we have obtained earlier (2.3) [6].

If to choose sign «–» we obtain solution for the case of magnetic intrinsic field and its velocity V_m:

$$E = BV_m, \qquad (10.33)$$

whence

$$cB = \frac{c}{V_m}E, \qquad (10.34)$$

what is in full accordance (without sign lost in scalar transformations) with equation (2.5) [6].

Equation (10.29) can be solved for magnetic field using three methods. It is possible to solve it by the same method as we have used concerning electric field, it is possible to do as has been done earlier and replace in (10.31) E by cB and vice versa. But it is also possible to do nothing because the equation (10.34) is ready solution for magnetic field, despite of that it is obtained proceeding from the equation (10.31) derived for electric field. If it is not known in advance, what field is intrinsic it is possible to use the equation (10.1) for invariant I_1 as has been described above.

10.6. Conclusion

Present paper shown that for moving fields which motion caused by motion of their source, classical equations for energy density are valid only at zero velocity of the source. These results are valid, in particular, for moving electron. If velocity of source and its field are nonzero the following formulas must be used:

- (10.19) or (10.20) - when source intrinsic field is known;

- (10.24) or (10.25) for total energy density when longitudinal and transverse components in laboratory frame of reference are known (in equation (10.24) magnetic field also taken into account implicitly, whereas equation (10.25) implicitly allows for electric field respectively).

Equation (10.27) is valid for energy flux of moving electric or magnetic field source. Energy flux in this case is equal to half of Poynting vector magnitude.

Generalizing the results obtained above for total energy, rest energy and kinetic energy of electromagnetic field and results of the previous papers of the present cycle concerning motion of electric and magnetic fields, it is possible to draw the following important conclusion: *all relativistic mechanics laws valid for substance, are valid also for electromagnetic field.*

Summary

1. By the example of charged electric capacitor it is shown that calculation using known formulas for electric and magnetic field total energy for moving fields leads to the results contradictory to energy and momentum conservation laws.

2. Equations for electric and magnetic field energy density for single moving field source are derived. These equations allow eliminate contradictions between classical electromagnetic field theory and the special relativity theory in calculation of electric and magnetic fields energy.

3. It is shown that when electric or magnetic field source moves perpendicular to field direction, the energy flux in laboratory frame of reference is equal to the Poynting vector multiplied by factor 1/2.

4. Expressions have been obtained that relates electric and magnetic field values to the field source intrinsic velocity.

5. It is shown that all main laws of relativistic mechanics that valid for substance are also valid for electromagnetic field.

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