The theory of electromagnetic field motion. 7. Electromagnetic field and charges

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In the present article general expressions for the electric and magnetic field divergence were obtained on the basis of the principles of the theory of electromagnetic field motion. Both the electric and magnetic bound charges were shown to exist in an inertial frame of reference. It was also shown that the rotating magnetic field causes a bound charge system to occur that represents as a whole the free electric charge. Important specific expressions for the inertial intrinsic frame of reference and the rotating magnetic field are considered and specific corresponding solutions were obtained. It was confirmed on the basis of the obtained general and specific expressions that the charges are not electromagnetic field sources, but are only its property.

7.1. Introduction

In the preceding works of the present cycle it has been shown that the magnetic field motion relative to observer causes the electric field to occur in a laboratory frame of reference. The electric field motion, in turn, causes occurrence of the magnetic field in a laboratory frame of reference. In those areas of electromagnetic field where flux lines appear or disappear, the field divergence is nonzero and, hence, it is necessary to speak about occurrence of electric or magnetic charges in this field area.

The electric charge appears not only when the magnetic field rotates as it was shown in [1], but also in straight-line motion of any magnetic field sources, in particular, of a current carrying loop that was considered in [2]. When a permanent magnet or a charged plane capacitor move, the open electromagnetic fields also appear, as it follows from Lorentz transformations for the electric and magnetic fields and, hence, field areas appear where the divergence of these fields is nonzero.

The question on electric or magnetic charge occurrence is avoided in the modern electromagnetic field theory, or actually ignored, in spite of the fact that Gauss's theorem directly states: at the points where the electric or magnetic field divergence is nonzero, electric or magnetic charges are located there respectively. Gauss's theorem is of fundamental character, closely related to Coulomb's law and is valid in any field where Coulomb's law holds true. It is for this reason Gauss's theorem is included in Maxwell's equations.

It is possible to single out two major factors by which the charges arising in the electric or magnetic field source motion are ignored. It is discontinuity between the classical electromagnetic theory and the special theory of relativity which is eliminated by the theory of electromagnetic field motion, and also by the concept on existence of «electric liquid» or other material carrier of a charge.

7.2. Divergence of electric and magnetic fields

Let's find a general expression for electric charge density, proceeding from the conclusions of the theory of electromagnetic field motion. With that end in view, we consider a case of a single magnetic field source. Then, in an intrinsic frame of reference of the source, inertial or noninertial, only the magnetic field is distinct from zero, that is the magnetic field of the source in the intrinsic frame of reference is, by definition in [2], its intrinsic field. Then, in the frame of reference related to the observer which also can be inertial or noninertial the intrinsic magnetic field velocity is equal to source velocity $\mathbf{V}_0$. Generally, $\mathbf{V}_0$ is a function of the coordinates and time. As appears from Maxwell's equations, electric charge density ρ_e is equal:

$$\rho_e = \varepsilon_0 \operatorname{div} \mathbf{E}, \tag{7.1}$$

Where ε_0 is an electric constant and $\mathbf{E}$ is the electric field strength in a laboratory frame of reference.

Let's find the electric field divergence by using expression (2.5) [2], which relates electric field $\mathbf{E}$ to magnetic field $\mathbf{B}$ in the laboratory frame of reference and to intrinsic magnetic field velocity $\mathbf{V}_{\mathrm{m}}$:

$$\operatorname{div} \mathbf{E} = \operatorname{div}[\mathbf{B}\mathbf{V}_{\mathrm{m}}], \tag{7.2}$$

whence it follows

$$\operatorname{div} \mathbf{E} = \mathbf{V}_{\mathrm{m}} \operatorname{rot} \mathbf{B} - \mathbf{B} \operatorname{rot} \mathbf{V}_{\mathrm{m}}. \tag{7.3}$$

It is considered in the classical electromagnetic field theory that because magnetic charges are absent the following equation holds:

$$\mathrm{div}\,\mathbf{B}=0. \quad (7.4)$$

If the magnetic charge is assumed to exist, the equality for magnetic charge density ρ_{m} must be valid:

$$\rho_{\mathrm{m}}=\varepsilon_0 c\,\mathrm{div}\,\mathbf{B}. \quad (7.5)$$

We have already established (it follows from Lorentz transformations for the electromagnetic field) that a source of magnetic flux lines can be not only magnetic charges, but also a moving electric field. Then, using expression (2.3) [2] for the case when the intrinsic field is the electric field, it is possible to write down:

$$\mathbf{B}=\frac{1}{c^2}\left[\mathbf{V}_{\mathrm{e}}\mathbf{E}\right]. \quad (7.6)$$

Here, as well as earlier, **E** is the electric field in the laboratory frame of reference, and $\mathbf{V}_{\mathrm{e}}$ is the intrinsic electric field velocity equal to the velocity of its source.

Let's find from (7.6) the magnetic field divergence:

$$\mathrm{div}\,\mathbf{B}=-\frac{1}{c^2}\mathbf{V}_{\mathrm{e}}\,\mathrm{rot}\,\mathbf{E}+\frac{1}{c^2}\mathbf{E}\,\mathrm{rot}\,\mathbf{V}_{\mathrm{e}}. \quad (7.7)$$

Analyzing expressions (7.3) and (7.7) we make use of their similarity, and use expression (7.3) as basic. As long as functions (7.3) and (7.7) are included into expressions (7.1) and (7.5) for electric and magnetic charge densities, while analyzing (7.3) and (7.7) we, in fact, mean the density of charges of the relevant fields.

Let's notice that the roles of the first and second terms in the right part of (7.3) and (7.7) are essentially different. To show this let`s again consider equation (7.3). We consider only two important special cases: when the intrinsic magnetic field frame of reference, in which the field is motionless, is inertial and when it is rotating and non-inertial. The second case is possible to express in other words: when the magnetic field rotates in a laboratory frame of reference.

7.3. The case of an inertial intrinsic frame of reference

The first case means that the intrinsic frame of reference related to the magnetic field source moves relative to the laboratory frame of reference rectilinearly without acceleration. The field of velocities of magnetic field $\mathbf{V}_m$ in this case is uniform and, hence, $\mathrm{rot}\ \mathbf{V}_m$ is equal to zero and the whole second term in the right part of (7.3) is equal to zero, too. In the case of the uniform intrinsic magnetic field the first term in (7.3) is also equal to zero because the rotor of the uniform field is zero. But in the case of the non-uniform field in these points where $\mathrm{rot}\ \mathbf{B}$ is nonzero, electric charges appear.

These charges are not related to any material carrier, except a moving magnetic field, but nevertheless, these are bound charges. They are bound to their source through the magnetic field, and their value depends on source velocity $\mathbf{V}_m$, as is obvious from (7.3). As an example we consider an almost closed magnet with a narrow flat gap, moving in the direction, perpendicular to the magnetic field direction in the gap. An electric field emerges in the gap as it moves, which is orthogonal both to the direction of motion, and to the magnetic field. Electric field flux lines originate at one edge of the gap, and come to an end on the other edge at the charges defined by (7.3). These charges are not discrete and are not quantized either in space or in magnitude, and are continuously distributed, with their density depending on the configuration and the magnetic field source velocity. At the points where the bound charges defined by the first term in the right part of expression (7.3) appear, the charge conservation law is locally violated. However, in other parts of the charge system related to the magnetic field source, there are electric charges of an opposite sign so the system total charge by summing over the entire volume occupied with the electromagnetic field of the moving source, will be equal to zero. It can be stated without any mathematical proof by using the charge conservation law, which holds in inertial reference frames.

From the positions described above let`s look once again at Feynman problem [3] concerning a current carrying wire and an electron moving over it. We have considered this problem earlier in [2]. Let`s return to this work and clarify the situation with the electric field arising near conductors and with the electric charges responsible for this field.

In the frame of reference related to the electron, an electric field appears which, Feynman explains by relativistic reduction in length of a wire, due to which the electron concentration increases and the electric field unbalanced by positive charges appears. But we noticed that in the presence of sliding contacts in the electric loop, in which the wire should be included and which is meant by default, the wire can move at any velocity, without influencing the electric field magnitude. The electric field magnitude depends on the magnetic field velocity equal to the velocity of the source, i.e. the electric loop.

The electric charge density of the moving magnetic field of the loop can be obtained from expression (7.3), moreover, as well as in the case with charges at magnet gap edges, arising charges are a property of the moving magnetic field and in no way are connected with the quantity of any charge carriers, electrons or ions. As well as in the case of a magnet, the moving magnetic field of the electric loop creates electric charges of an opposite sign so the total charge appears equal to zero at the opposite side of the loop.

What was said above for expression (7.3) concerns also expression (7.7) for the magnetic field divergence. In particular, if a charged flat capacitor moves orthogonally to the electric field direction, there will be a magnetic field in the capacitor volume according to Lorentz electromagnetic field transformations, and at the edges of the capacitor magnetic charges emerge according to (7.7).

These are bound magnetic charges and they have nothing in common with the monopole. Nevertheless, if the areas where magnetic flux lines originate and where they come to an end are separately surrounded by closed surfaces, it will appear that for both the areas singularly, locally, the magnetic field induction divergence is nonzero. When the areas are combined so as to cover the capacitor field entirely, the flux of the total magnetic induction vector will obviously be equal to zero.

7.4. The case of a rotating magnetic field

Let's return to the equation (7.3) and consider the second case when the laboratory frame of reference or the magnetic field rotates. From the point of view of the analysis done for expression (7.3) these cases are equivalent and differ only in a velocity sign. We are really interested in the rotating field only, so for the sake of clarity it is this case that is to be considered.

Let's find rot $\mathbf{V}_m$ for uniform rotation of an intrinsic frame of reference at angular velocity. We believe that the intrinsic magnetic field rotates as a whole, like a rigid body.

By omitting index m at vector $\mathbf{V}_m$ an, we can write down:

$$\mathbf{V} = [\omega\mathbf{r}], \tag{7.8}$$

where $\mathbf{r}$ is a position vector.

Let's choose cylindrical coordinate system ρ, φ, z and write down the expression of the velocity rotor for the system in its general form [4]:

$$\operatorname{rot}\mathbf{V} = \left(\frac{1}{\rho}\frac{\partial V_z}{\partial \varphi} - \frac{\partial V_\varphi}{\partial z}\right)\mathbf{i}_\rho + \left(\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial \rho}\right)\mathbf{i}_\varphi + \left[\frac{\partial(\rho V_\varphi)}{\partial \rho} - \frac{\partial V_\rho}{\partial \varphi}\right]\mathbf{i}_z, \tag{7.9}$$

where $\mathbf{i}_\rho$, $\mathbf{i}_\varphi$, $\mathbf{i}_z$ are unit vectors. We use symbol ρ without an index, unlike the charge density, as one of the coordinates of the cylindrical coordinate system.

For the ρ, φ, z coordinate system with axis z parallel to vector $\boldsymbol{\omega}$, equation (7.8) will become:

$$V_\rho = 0, \tag{7.10}$$

$$V_\varphi = \omega\,\rho, \tag{7.11}$$

$$V_z = 0. \tag{7.12}$$

In substitution of the obtained values in (7.9) the first two terms and the second component of the last term in expression (7.9) become zero. After differentiation and transition to a vector form, we obtain the expression known from the theory of rotation of a rigid body:

$$\mathrm{rot}\,\mathbf{V} = 2\boldsymbol{\omega}. \tag{7.13}$$

Then by substituting (7.13) and (7.8) in (7.3) for the case of the rotating magnetic field at angular velocity **ω,** we obtain:

$$\mathrm{div}\,\mathbf{E} = \left[\omega\mathbf{r}\right]\mathrm{rot}\,\mathrm{B} - 2\,\omega\mathrm{B}\,. \tag{7.14}$$

The first term in (7.14) may be equal to zero as well as be nonzero, depending on the specific field configuration. As long as **ω** is a constant value, the second term in (7.14) is completely defined by the configuration of magnetic field **B**, hence, the contribution of the second term to the total charge density is also defined by configuration of field **B**. All said above relates to any limited area of space if function **B** at the borders of the area and its partial derivatives are continuous.

Equation (7.14) is of special importance in development of the theory of electron because it holds, unlike (6.28) [1], not only for a point dipole, but also for any configuration of a rotating magnetic field with axial symmetry. In particular, this relates to the electron core where, the magnetic field configuration obviously cannot correspond to a point dipole. For this reason let`s consider some important special cases of expression (7.14).

7.5. Important special cases of equation (7.14)

1. The case of a uniform magnetic field in some area and in its nearest vicinity.

In this case the first term in (7.14) is equal to zero because rotor **B** is equal to zero, and the second term is a constant value. Hence, according to (7.1), the electric charge density in the considered area is homogeneous.

2. The case of axially-symmetric field with the axis of rotation coinciding with the axis of symmetry, and field component $B_\varphi = 0$ in the spherical and cylindrical coordinate systems.

Let's write down rotor of vector **B** in spherical coordinate system r, ϑ, φ [4]:

$$\operatorname{rot}\mathbf{B} = \frac{1}{r\sin\vartheta}\left[\frac{\partial\left(B_\varphi \sin\vartheta\right)}{\partial\vartheta} - \frac{\partial B_\vartheta}{\partial\varphi}\right]\mathbf{i}_r + \frac{1}{r}\left[\frac{1}{\sin\vartheta}\frac{\partial B_r}{\partial\varphi} - \frac{\partial\left(rB_\varphi\right)}{\partial r}\right]\mathbf{i}_\vartheta +$$

$$+\frac{1}{r}\left[\frac{\partial\left(rB_\vartheta\right)}{\partial r} - \frac{\partial B_r}{\partial\vartheta}\right]\mathbf{i}_\varphi, \tag{7.15}$$

where $\mathbf{i}_r, \mathbf{i}_\vartheta, \mathbf{i}_\varphi$ are unit vectors.

By virtue of the fact that $B_\varphi = 0$ and the magnetic field has axial symmetry, all partial derivatives entering into first two terms (7.15) are also equal to zero and, hence, only the third term in expression (7.15) is nonzero. By substituting (7.15) in (7.14), taking into account the above mentioned, we obtain:

$$\operatorname{div}\mathbf{E} = \frac{1}{r}\left[\frac{\partial\left(rB_\vartheta\right)}{\partial r} - \frac{\partial B_r}{\partial\vartheta}\right]\left[\boldsymbol{\omega}\mathbf{r}\right]\mathbf{i}_\varphi - 2\boldsymbol{\omega}\mathbf{B}. \tag{7.16}$$

3. The case of the cylindrical field, coinciding by configuration with the magnetic field in the central part of the long solenoid whose axis coincides with the rotation axis. There is considered a cylindrical area of the field uniform by the length, but not necessarily uniform in radial direction. Case 3 is really specific in relation to case 2, i.e. expression (7.16) holds for it. The spherical coordinate system, however, in this case is less convenient in comparison to cylindrical.

Let's proceed to cylindrical coordinate system ρ, φ, z. To do it more easily is possible on account of the general expression for the rotor of magnetic field induction in coordinate system ρ, φ, z [4]:

$$\operatorname{rot}\mathbf{B} = \left(\frac{1}{\rho}\frac{\partial B_z}{\partial\varphi} - \frac{\partial B_\varphi}{\partial z}\right)\mathbf{i}_\rho + \left(\frac{\partial B_\rho}{\partial z} - \frac{\partial B_z}{\partial\rho}\right)\mathbf{i}_\varphi + \frac{1}{\rho}\left[\frac{\partial\left(\rho B_\varphi\right)}{\partial\rho} - \frac{\partial B_\rho}{\partial\varphi}\right]\mathbf{i}_z. \tag{7.17}$$

The first and third terms in (7.17) are equal to zero for the same reasons, as in expression (7.15). So long as in the considered cylindrical area field component B_ρ does not depend on ρ, the first component in the second term is also equal to zero. Then, by substituting (7.17) in (7.14) we obtain:

$$\operatorname{div}\mathbf{E} = -\frac{\partial B_z}{\partial \rho}[\boldsymbol{\omega}\mathbf{r}]\mathbf{i}_\varphi - 2\boldsymbol{\omega}\mathbf{B}. \tag{7.18}$$

Case 1 considered earlier can be considered as specific in relation to case 3 if the cylindrical area is chosen in a uniform field. So long as the field is uniform, the first term in expression (7.18) will be equal to zero and we come to the same conclusions, as in case 1.

7.6. Electric and magnetic charges in the theory of electromagnetic field movement

Let's note one important circumstance due to the character of equation (7.3) and following from the analysis of the results obtained in aricle [1] and from consideration of special cases in the present work.

The second term in equation (7.3), and also in its special cases (7.14), (7.16), (7.18) results in, as well as the first term, occurrence of bound charges. However, unlike the first term, these charges may not be compensated by charges of an opposite sign, as well as it was in all the cases considered above. This leads to the fact that in the case of a rotating laboratory frame of reference, as shown in [1] and as appears from (7.3), (7.14), (7.16), (7.18), if these expressions are used in a non-inertial rotating laboratory frame of reference, the charge conservation law is not observed. At the same time, in the inertial laboratory frame of reference the charge conservation law as it is known is valid. It means that the mechanism, which can force the magnetic field to rotate or to stop, is not only unknown, but also is essentially impossible. In the microworld, charged elementary particles can emerge, but only in pairs, so the total charge remains invariable.

Expression (7.7) for magnetic charges differs from expression (7.3) for electric charges by the fact that in the inertial frame of reference the second term in (7.7) is always equal to zero. This follows from the experimentally established fact that there is no magnetic monopole in the nature. Hence, the rotor of electric field velocity in inertial frame of reference is always equal to zero. There are no rotating electric fields in the nature. Preservation of the second term in expression (7.7) is justified only if (7.7) is supposed to be applied in the non-inertial frame of reference. For the inertial frame of reference, as appears from (7.7), the following equation is valid:

$$\operatorname{div}\mathbf{B} = -\frac{1}{c^2}\mathbf{V}_\mathrm{e}\operatorname{rot}\mathbf{E}. \tag{7.19}$$

In summary let's make some remarks concerning a role of the very charge concept in the electromagnetic field theory.

Since Faraday times the electric charge was believed to be an electric field source, whereas the electric current was believed to be a magnetic field source. Generalizing the results obtained in present and in the preceding works of the cycle, it is possible to conclude that *in every case presence of any electromagnetic field component, electric or magnetic, is due to motion of another component, and electric or magnetic charges are just an electromagnetic field property.*

The fact that the charge is just a property of the electromagnetic field, not its source, does not belittle at all the charge's role as a physical concept. There are two interconnected causes. Firstly, all the mathematical apparatus of electromagnetic field theory is based on this concept. Secondly, a huge set of problems are effectively solved using the charge concept as a field source. It may be imaged how many work had to be done to calculate interaction force between two charges on the basis of energy calculation in every point of the electric field of interacting charges, and how simply it can be made by using Coulomb's law. Therefore, in spite of the fact that the source of charges is the electromagnetic field, and not the reverse, the role of the electric or a magnetic field concept remains in the electromagnetic theory in full.

Conclusions

1. General expressions for the electric and magnetic field divergence are obtained on the basis of the concepts that in all the cases occurrence of

any component of the electromagnetic field, electric or magnetic, is caused by motion of another component.

2. Both the electric and magnetic bound charges were shown to exist in the inertial frame of reference.

3. It was noticed that the rotating magnetic field causes occurrence of a system of bound charges which is, as a whole, a free electric charge. Also it is noticed that in the nature there are no rotating electric fields because the magnetic monopole has not been found out experimentally.

4. Special cases of an inertial intrinsic frame of reference and a rotating magnetic field were considered. The relevant specific equations were obtained for these cases.

5. It was noticed that charges are not a source of the electromagnetic field, but are only its property.

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